



Allocating “Risk Dollars” Back to Individual Cost Elements

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Contents

- **What Are “Risk Dollars”?**
 - What is Your “Point” Estimate?
 - What Level of Confidence Do You Need?
- **Why Allocate Risk Dollars?**
 - The Political Reason
 - The Project-Management Reason
- **How Should We Allocate Risk Dollars?**
 - The Difference between Uncertainty and Risk
 - How Many Risk Dollars Does Each WBS Element Need?
- **Summary**

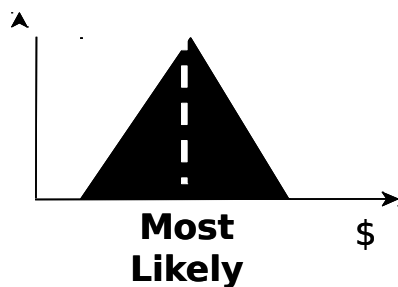
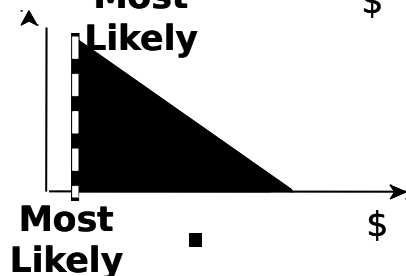
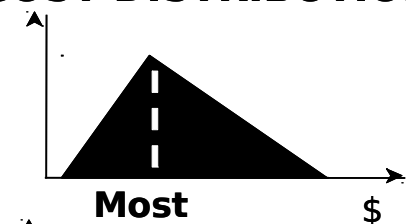


The Term "Point Estimate" Must be Formally Defined

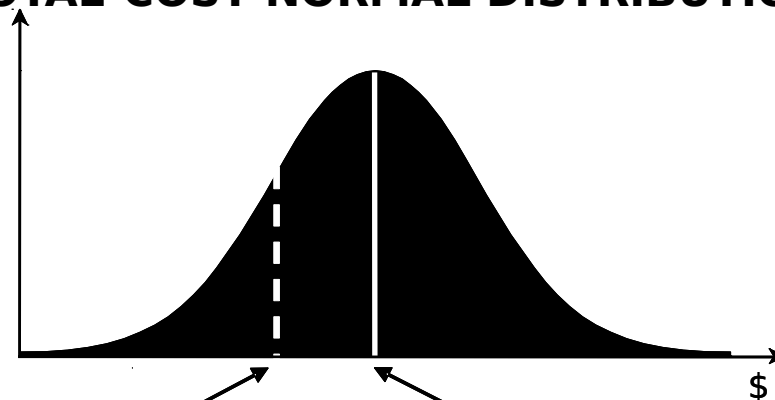
- **A Necessity if the “Point” Estimate is to Serve as a Basis to which “Risk Dollars” will be Appended**
- **By “Point” Estimate, Do You Mean the ...**
 - ... “Most Likely” Cost? (“Mode”)
 - ... 50th-Percentile Cost? (“Median”)
 - ... Expected Cost? (“Mean”)
 - ... the “Roll-Up” of Most Likely Costs of Each WBS Element?
 - ... Something Else?
- **When Estimating Costs of Complex Hardware and/or Software Systems, These Numbers are Almost Always Different (Especially the “Something Else”)**
- **To Illustrate the Ideas, This Discussion Will Consider the “Point” Estimate to be the Roll-Up (i.e., “Sum”) of the WBS-Element Most Likely Costs**

The Roll-Up "Point Estimate" in Pictures

WBS-ELEMENT TRIANGULAR COST DISTRIBUTIONS



MERGE WBS-ELEMENT COST DISTRIBUTIONS INTO TOTAL-COST NORMAL DISTRIBUTION*



ROLL-UP OF MOST LIKELY
WBS-ELEMENT COSTS

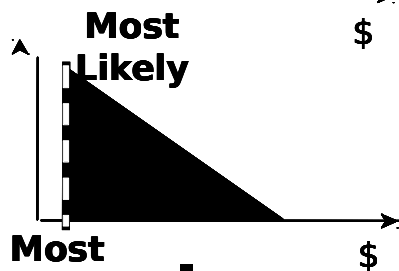
MOST LIKELY
TOTAL COST

Note: The roll-up of WBS element most likely costs is not equal to the most likely total cost.

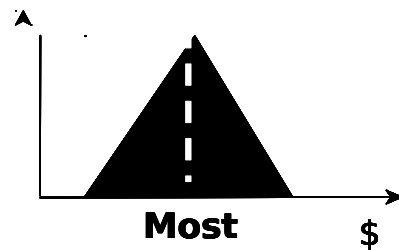
* When the Number of WBS Elements is "Large"

When WBS Elements Are Few...

WBS-ELEMENT TRIANGULAR COST DISTRIBUTIONS

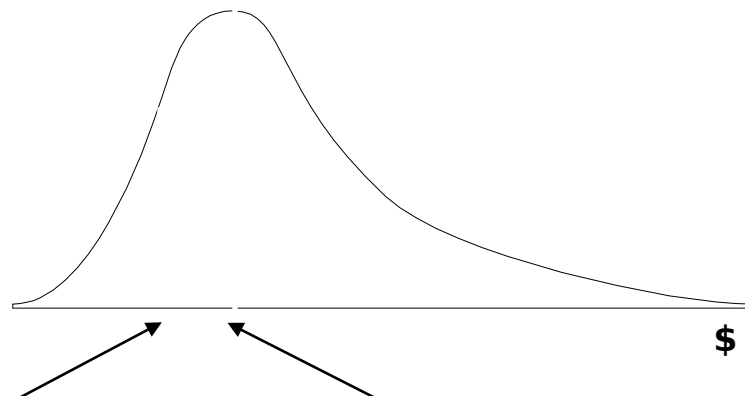


**Most
Likely**



**Most
Likely**

MERGE WBS-ELEMENT COST DISTRIBUTIONS INTO TOTAL-COST LOGNORMAL DISTRIBUTION



**ROLL-UP OF MOST LIKELY
WBS-ELEMENT COSTS**

**MOST
LIKELY
TOTAL
COST**

**Note: The roll-up of WBS element most likely costs
is still not equal to the most likely total cost.**

"Risk Dollars"

- **Amount of Additional Dollars (beyond the Point Estimate) Required to Fund Program at an Appropriate Level of Confidence**
 - If Point Estimate is the Roll-up of Elements' Most Likely Costs, Lots of Risk Dollars Will Be Needed to Reach 50% Confidence Level and Even More to Reach 65% or 70% Level
 - If Point Estimate is the Most Likely Total Cost, Some Additional Risk Dollars Will Usually Be Required to Reach 50% Confidence Level and Some More Will be Needed to Reach the 65% or 70% Level
 - If Point Estimate is the 50th-percentile Cost, Additional Risk Dollars Will Be Required to Reach the 65% or 70% Confidence Level
 - If Point Estimate is the Expected Cost, Additional Risk Dollars Will Be Required to Reach 65% or 70% Level
 - Point Estimates Can be Selected for other Characteristics
- **Sometimes Called ...**
 - Management Reserve
 - (Sneeringly) "Slush Fund"

Why Allocate?

- **Your Request to Funding Authority**
 - **"Our Point Estimate is \$ΩM, but We Also Foresee a Need for an Additional \$ΘM to Cover Identified Technical and Programmatic Risks"**
- **Common Responses from Funders**
 - **"What? Don't You Know How Much Your Program is Going to Cost?" - "Do You Even Know How You are Going to Manage the Program?"**
 - **"That's a Rather Large Slush Fund - What Are You Going to Do With It?"**
- **Your Answer**
 - **"We are Pushing the State of the Art in a Number of Technology and Software Areas and There are Several Other Risk Issues Due to the Innovative Nature of This Program. I'll Show You How We Plan to Allocate the Additional Funds to Manage the Various Risk Issues and Make Our Program Executable."**



How Will the Risk Dollars *Actually* Be Spent?

- **Not the Way You Think**
 - After All, They're *Risk* Dollars
 - They'll Be Spent on Risks that Turn Out to Be Critical
- **All the Risk Dollars Must be Retained by the Program Manager Until Specific Need Materializes**
 - That's Why it's Often Referred to as "*Management Reserve*"
- **Then Why are We Doing the Allocation Now?**
 - We're Not *Really* Allocating the Money Now
 - We're Merely Proposing that Some (or All) WBS Elements May Need Extra Money in Proportion to Their Riskiness
 - We are Making that Extra Money Part of our Cost Estimate
 - "The Race Is Not to the Swift, nor the Battle to the Strong, ..." (*Ecclesiastes*, 9:11), but That's the Way to Bet



“Roll-Up” Issues Impact Risk-Dollar Allocation Method

- **Mathematical Facts**
 - **Most-Likely Project-Element Costs Do Not Sum to Most Likely Total Project Cost**
 - **n th Percentiles of Project-Element Costs Do Not Sum to n th Percentile of Total Project Cost**
 - **If Project-Element Costs are Correlated, the Correlations Must be Taken into Account when Summing Element Costs (usually by Monte Carlo) to Obtain the Distribution of Total Project Cost**
- **These Mathematical Facts Guarantee that There is no Simple Way (or even a Unique Right Way) to Allocate Risk Dollars Back to the WBS Elements**

To Allocate the Risk Dollars ...

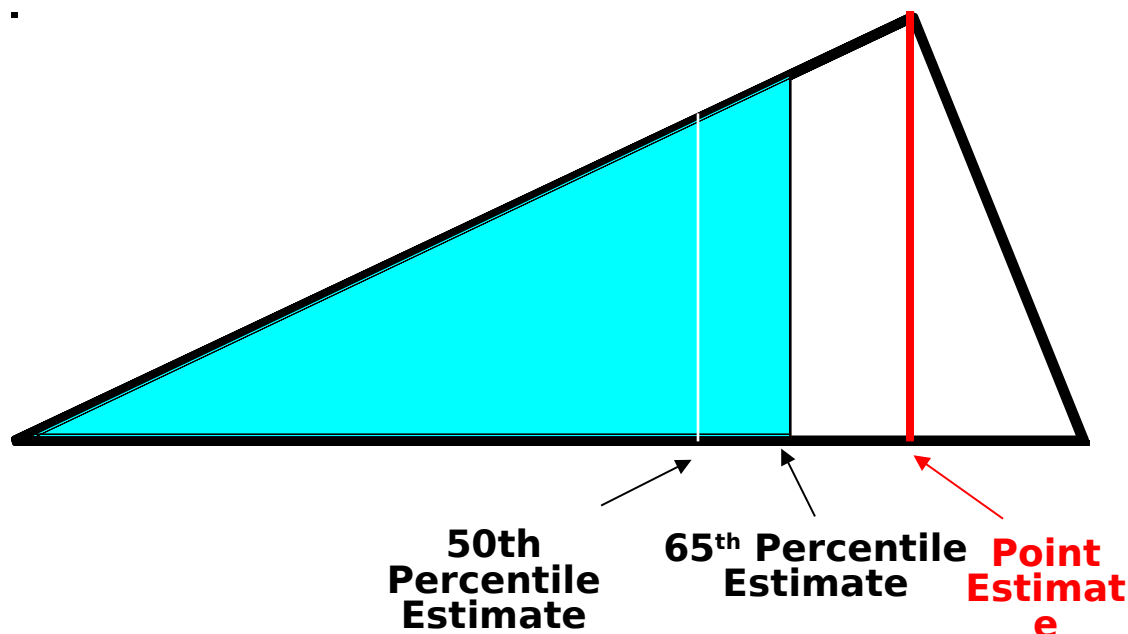
- **Calculate Total Amount of Risk Dollars Required**
 - **50thRisk\$ = 50th-percentile Total Cost, Minus Roll-Up; or**
...
 - **65thRisk\$ = 65th-percentile Total Cost, Minus Roll-Up**
 - **70thRisk\$ = 70th-percentile Total Cost, Minus Roll-Up**
 - **Of Course, Other Percentiles (80th, 90th, etc.) May be Considered Appropriate for a Particular Project, as Well as Other Definitions of the Point Estimate**
- **Allocation of Risk Dollars to Project Elements Must Put Risk Dollars Where They are "Needed"**
- **We Must Define an Element's "Need" in Order to Determine How Many Risk Dollars are "Needed"**
 - **Need_{*j*} = Dollar "Need" of Project-element *j* (to Be Defined Precisely Later)**
 - **Corr_{*ij*} = Correlation Between Risk-dollar Requirements of Project Elements *i* and *j***



Our Specifications on the Risk-Dollar Allocation Procedure

- 1. Those WBS Elements Having More Cost Risk Shall be Allocated More Risk Dollars, Relative to their Point Estimates**
- 2. Inter-Element Correlation Shall be Taken into Account when Calculating an Element's Risk-Dollar Allocation**
 - Correlated Elements Shall "Share" Risk Dollars**
 - Risk Dollars Shall not be "Double-Allocated"**
- 3. The Risk-Dollar Allocation Shall Not Result in a WBS-Element's Estimate Being Reduced Below its Point Estimate**
 - This Means that Risk Dollars Shall Not be Subtracted from a Point Estimate**
 - Therefore the Fewest Possible Number of Risk Dollars Allocated to any WBS Element will be Zero**

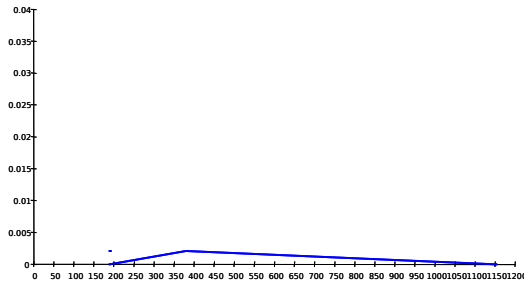
Impact: No Element's Estimate is Reduced Below its Point Estimate



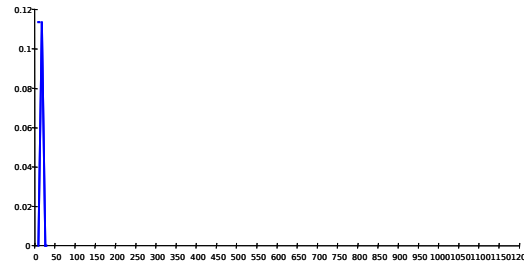
- **"The Risk-Dollar Allocation Shall Not Result in a WBS-Element's Estimate Being Reduced Below its Point Estimate" Even if ...**
 - The Point Estimate Exceeds the 50th Percentile Estimate
 - The Point Estimate Exceeds the 65th Percentile Estimate
 - The Point Estimate Exceeds the 90th Percentile Estimate



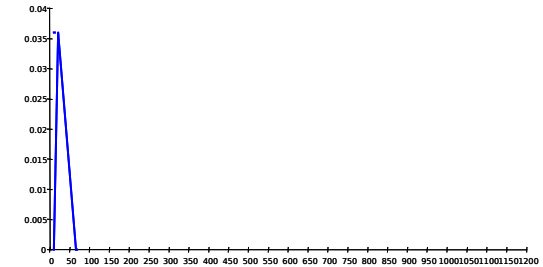
Example: System X WBS-Element Triangular Cost Distributions



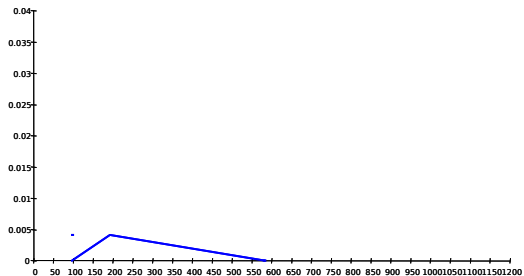
1. Antenna



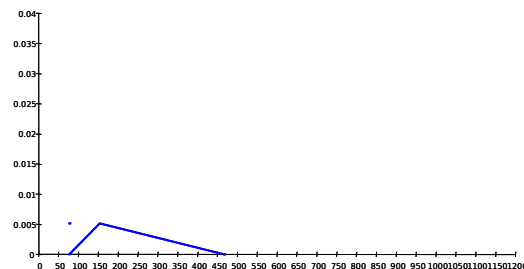
4. Facilities*



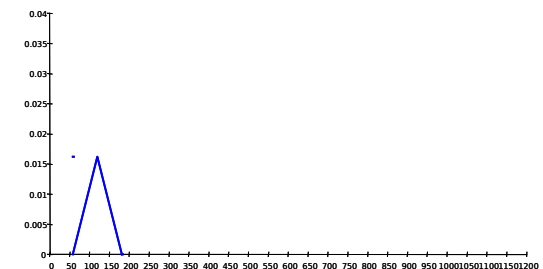
7. Environmental Control



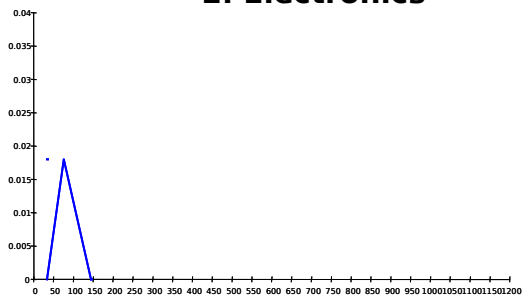
2. Electronics



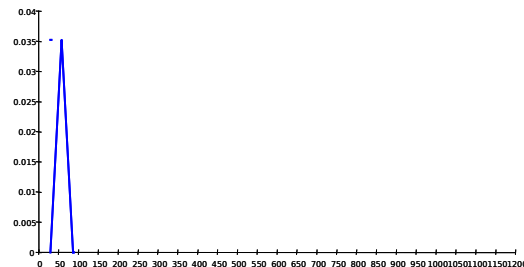
5. Power Distribution



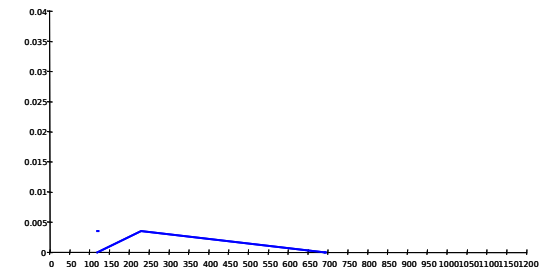
8. Communications



3. Platform



6. Computers



9. Software

* Vertical scale (probability density) different in this graph only.

System X Triangular Distributions

WBS Element	L	M	H	Mean**	Sigma
1. Antenna	191	380	1151	574	207.62
2. Electronics	96	192	582	290	105.08
3. Platform	33	76	143	84	22.63
4. Facilities	9	18	27	18	3.67
5. Power Distribution	77	154	465	232	83.86
6. Computers	30	58	86	58	11.43
7. Environmental Control	11	22	66	33	11.88
8. Communications	58	120	182	120	25.31
9. Software	120	230	691	347	123.68
SUMS		1250*		1756**	

* Point Estimate (Not the Same as the Most Likely Total Cost)

** "Mean" = "Expected Cost" (Note: Sum of WBS-Element Means is Equal to the Total-Cost Mean.)

System X Inter-Element Correlations

Correlation Matrix

		WBS Element								
		1	2	3	4	5	6	7	8	9
WBS Element	1	1.00	0.50	0.50	0.60	0.50	0.50	0.30	0.70	0.70
	2		1.00	0.40	0.50	0.50	0.60	0.50	0.50	0.70
	3			1.00	0.70	0.60	0.70	0.70	0.50	0.70
	4				1.00	0.40	0.40	0.50	0.30	0.60
	5					1.00	0.50	0.50	0.50	0.70
	6						1.00	0.40	0.70	0.80
	7							1.00	0.50	0.70
	8								1.00	0.80
	9									1.00

Computation of Total Cost Standard Deviation (σ)

WBS Element	L	M	H	Mean**	Sigma
1. Antenna	191	380	1151	574	207.62
2. Electronics	96	192	582	290	105.08
3. Platform	33	76	143	84	22.63
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SUMS		1250*		1756	

		WBS Element								
		1	2	3	4	5	6	7	8	9
WBS Element	1	1.00	0.50	0.50	0.60	0.50	0.50	0.30	0.70	0.70
	2		1.00	0.40	0.50	0.50	0.60	0.50	0.50	0.70
	3			1.00	0.70	0.60	0.70	0.70	0.50	0.70
	4				1.00	0.40	0.40	0.50	0.30	0.60
	5					1.00	0.50	0.50	0.50	0.70
	6						1.00	0.40	0.70	0.80
	7							1.00	0.50	0.70
	8								1.00	0.80
	9									1.00

**Use the WBS-Element "Sigma" Values
and the Inter-Element Correlations to**

Compute

$$\sigma_T = \sqrt{\sum_{j=1}^n \sigma_j^2 + 2 \sum_{j=2}^n \sum_{i=1}^{j-1} \rho_{ij} \sigma_i \sigma_j} = 491.78$$

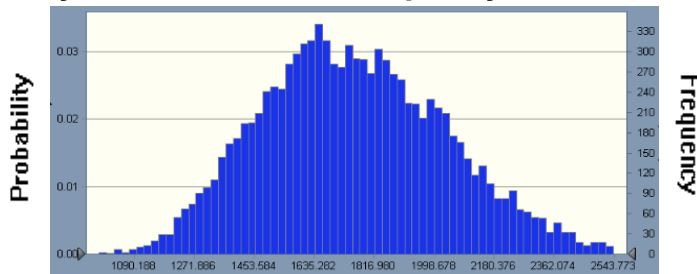


System X Total-Cost Percentiles (as Output by Crystal Ball®)

Monte Carlo Statistics	Crystal Ball Output Values
Trials	10,000
Mean	1,756.00
Median	1,739.86
Point Estimate	1,250.00
Standard Deviation	280.39
Coeff. of Variability	0.16
Range Minimum	983.67
Range Maximum	2,783.98
Range Width	1,800.31

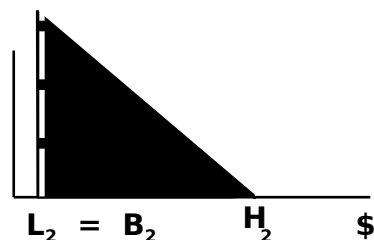
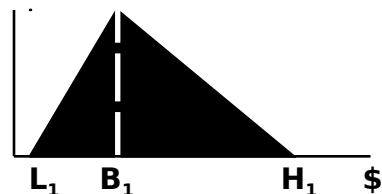
Percentile (Confidence Level)	Cost Value
5%	1,320.16
10%	1,399.64
15%	1,460.09
20%	1,508.89
25%	1,554.15
30%	1,592.97
35%	1,629.75
40%	1,662.51
45%	1,700.33
50%	1,739.86
55%	1,777.18
60%	1,818.64
65%	1,857.85
70%	1,898.94
75%	1,947.32
80%	1,999.56
85%	2,054.81
90%	2,130.96
95%	2,244.74

System X Total-Cost Frequency Distribution

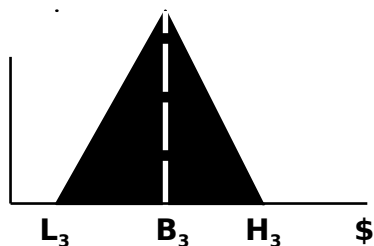


Risk-Dollar Definition

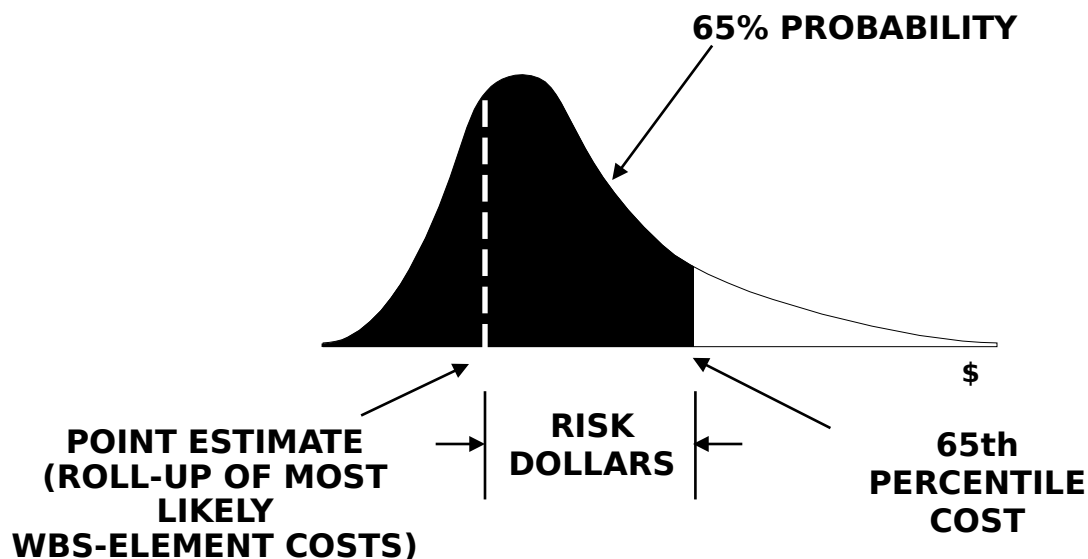
COST-ELEMENT TRIANGULAR DISTRIBUTIONS



⋮



MERGE ELEMENT COST DISTRIBUTIONS INTO TOTAL-COST DISTRIBUTION



Notes: 1. Addition of risk dollars increases confidence that total estimate (point estimate plus risk dollars) is sufficient to execute program.
2. Point estimate may also be chosen to be the 50th Percentile ("Median"), Expected Cost ("Mean"), or Most Likely Cost ("Mode"), as well as the "Roll-Up" Estimate.

Statistical Facts About Risk

- **Impact of Risk on Cost is Modeled as "Uncertainty" in Cost**
- **Uncertainty in Cost Means That Cost Distributions Range Over Wide Intervals**
 - More Uncertainty Means a Wider Range
 - Less Uncertainty Means a Narrower Range
- **Sigma (σ), or Standard Deviation, the Statistical Measure of Range of Cost Distribution, Is Universal Measure of Uncertainty**
- **Question: Is σ a Good Measure of Risk?**

How Does σ Measure Uncertainty?

$\forall \sigma^2$ is the Mean (i.e., "Average") Squared Distance from the Mean of the Distribution

- $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$ where

μ = Mean ("Expected Value") of Distribution

$f(x)$ = Probability Density Function

- If Inter-Element Correlations are Zero

$$\sigma_{TOTAL}^2 = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2 = \sum_{k=1}^n \sigma_k^2$$

- If ρ_{ij} = Correlation Between Project Elements i and j

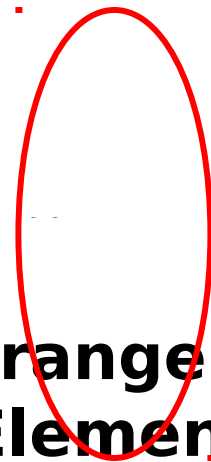
$$\sigma_{TOTAL}^2 = \sum_{k=1}^n \sigma_k^2 + 2 \sum_{j=2}^n \sum_{i=1}^{j-1} \rho_{ij} \sigma_i \sigma_j$$



Algebraic Analysis of the Total Cost Standard Deviation

- **Larger σ Implies More Uncertainty, Which in Turn May Imply Greater Need for Risk Dollars**
- **Consider the Following Algebraic Rearrangement of the Total-Cost σ Value:**

- **Let's See What the Algebraic Rearrangement Means in the Case of $n = 4$ WBS Elements**

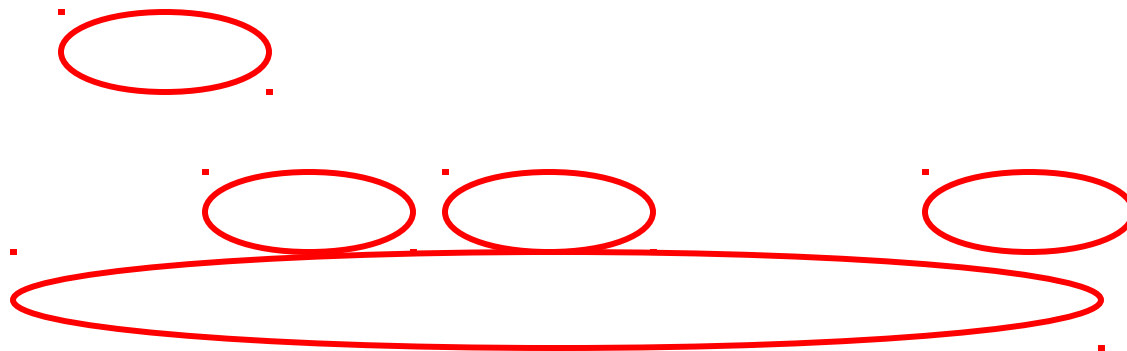




Total Cost Standard Deviation for $n = 4$ WBS Elements, Chart 1 of 3



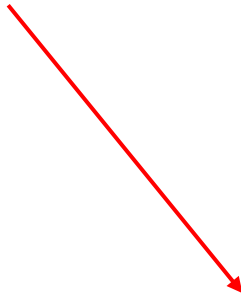
Total Cost Standard Deviation for $n = 4$ WBS Elements, Chart 2 of 3





Total Cost Standard Deviation for $n = 4$ WBS Elements, Chart 3 of 3

**Portion
of
Total
Cost
 σ^2 that is
Associate
d
With
WBS
Element j**

A red arrow originates from the right side of the text box and points diagonally downwards and to the right, towards the center of the slide.

Try This: Use σ Values to Allocate Risk Dollars Based on Uncertainty

- **Consider the Following n -Element Version of the Representation of the Total-Cost σ^2 Value, the 4-Element Version of which Appears on the Previous Chart:**

$$\sigma_{TOTAL}^2 = \sum_{j=1}^n \sum_{i=1}^n \rho_{ij} \sigma_i \sigma_j$$

- **The Portion of the Total-Cost σ^2 Value that is Associated, either Directly or via Correlation, with WBS Element k is Given by the Following Expression:**

$$\sigma_{ASSOC(k)}^2 = \sum_{i=1}^n \rho_{ik} \sigma_i \sigma_k$$

σ -Based Allocation Formula

- **Uncertainty Base** $\sigma_{TOTAL}^2 = \sum_{j=1}^n \sum_{i=1}^n \rho_{ij} \sigma_i \sigma_j$
- **Fraction of Risk Dollars to be Allocated to Element k Should Therefore be:**

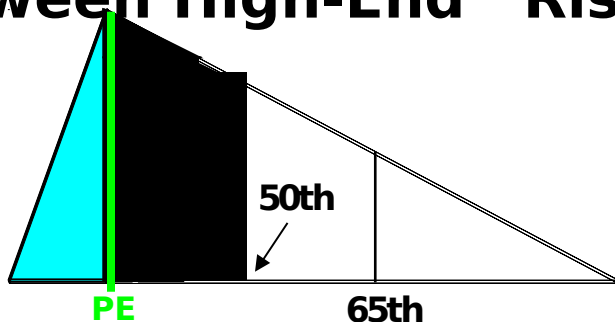
$$\frac{\sigma_{ASSOC(k)}^2}{\sigma_{TOTAL}^2} = \frac{\sum_{i=1}^n \rho_{ik} \sigma_i \sigma_k}{\sum_{j=1}^n \sum_{i=1}^n \rho_{ij} \sigma_i \sigma_j} = \frac{\text{Uncertainty Portion for } k}{\text{Uncertainty Base}}$$

- **Amount of Risk Dollars to be Allocated to Element k is Therefore**

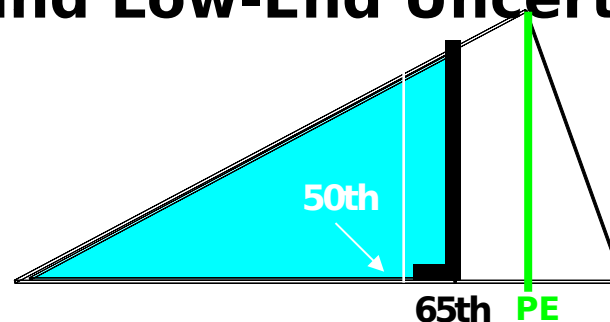
$$\frac{\sum_{i=1}^n \rho_{ik} \sigma_i \sigma_k}{\sum_{j=1}^n \sum_{i=1}^n \rho_{ij} \sigma_i \sigma_j} \times \text{Total Amount of Risk Dollars}$$

Now for the Bad News

- **Unfortunately, This Very Clever Procedure is No Good***
- **σ as a Measure of Uncertainty Cannot Distinguish Between High-End "Risk" and Low-End Uncertainty**



vs.



- **Both Distributions Have the Same σ Value, but the One on the Left (with the High-End Risk) Needs Lots of Risk Dollars to Reach its 50th Percentile and Even More to Reach its 65th Percentile**
- **The One on the Right is More than Fully Funded to its 65th Percentile by its Point Estimate (PE)**

*Of course, that won't stop people from using it!

This Problem with σ is Not New

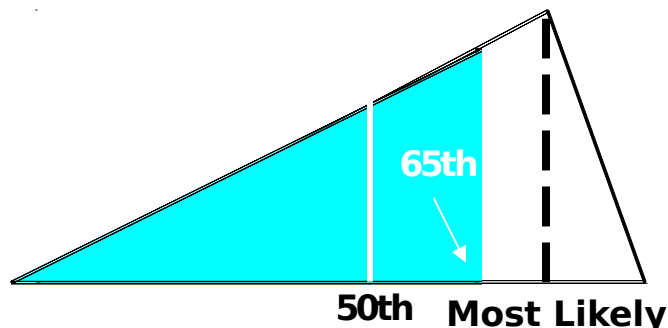
- **C.-C. Ho and W. Chen (Internal Revenue Service), “Selected Semi-Variance Estimators of Underreporting Nonfarm Sole Proprietor Income,” *Proceedings of the American Statistical Association*, Section 28, 1996, pages 198-203.**
 - “Full variance considers extremely high and extremely low underreported receipts or overstated expenses equally undesirable. Semi-variance, on the other hand, measures deviations from the mean for observations below or above the mean.” (page 198)
 - “We extend the semi-variance estimators described above to a covariance context and develop a semi-variance based correlation coefficient between a pair of a selected half of X and a selected half of Y” (page 199)
- **B.J. Jacobsen (Chief Economist, Capital Markets Consultants, Milwaukee WI), “Mean-Semi-Variance Efficient Frontier,” i+7 pages.**

www.cmarkc.com/pdf/cmc_mean_semi_variance_efficient_frontier.pdf

- “...This result is critically dependent on investor utility being a function of the standard deviation (or variance) of returns. But why should risk be defined in such a way? Introspection would suggest that investors are primarily concerned about losing money, not making money. To take this behavioral consideration into account, ...we found the minimal semi-standard deviation of returns.” (pages 2-3)

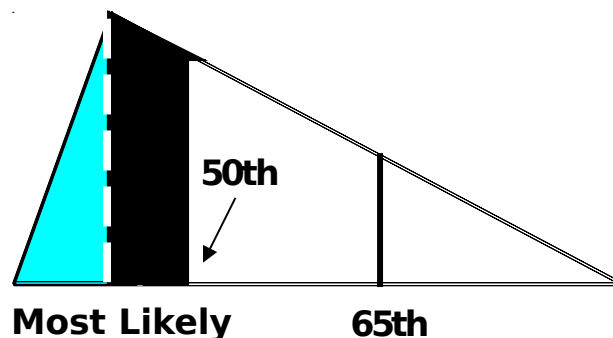
Let's Look at the "Need" of WBS Element k for Risk Dollars

- Calculated "Need" of Any WBS Element is Based on its Probability of Overrunning its Point Estimate (which here is its "Most Likely Cost" = Mode)
- An Element that Has Preponderance of Probability *Below* its Point Estimate (such as the distribution on the left) Has Little or No Need
- Proposed Definition of "Need" of Project Element k at the 80th Percentile Level
 - $\text{Need}_k = 65^{\text{th}}$ Percentile Cost Minus Point Estimate
 - $\text{Need}_k = 0$ If Point Estimate Exceeds 65th Percentile Cost



$\text{Need}_k =$

0



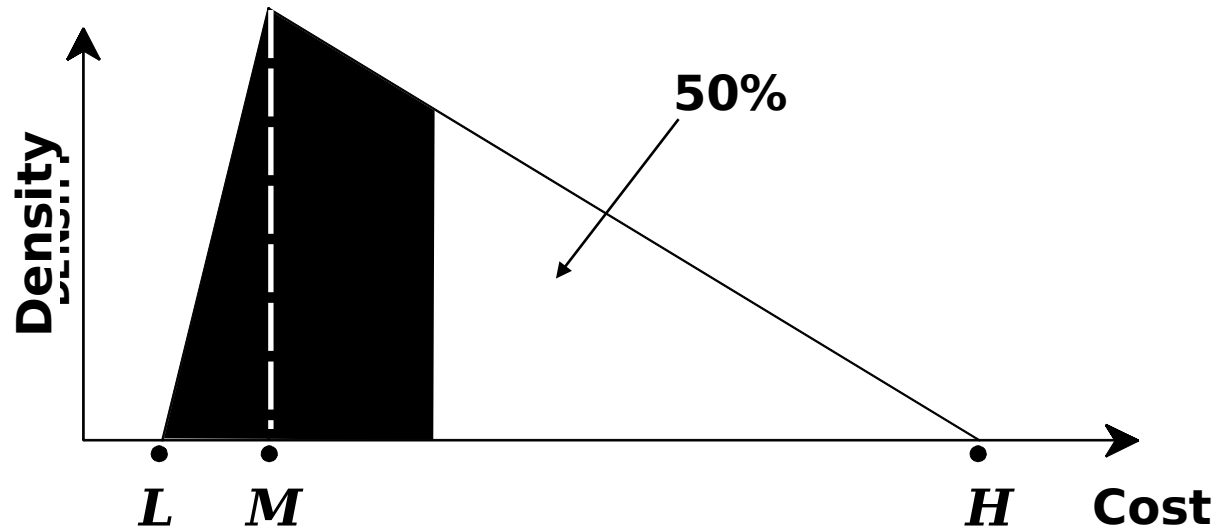
$\text{Need}_k >>$

0

Need-Based Allocation Formula

- **Total Need Base (Analogue of Total-Cost σ^2)**
 - Need Base $\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} \text{ Need}_i \text{ Need}_j$
- **Need "Portion" for Project Element k (Analogue of Portion of Total-Cost σ^2 that is Associated with Element k)**
 - $\sum_{i=1}^n \rho_{ik} \text{ Need}_i \text{ Need}_k$
- **Risk Dollars Allocated to Project Element k**
 - $= \left(\sum_{i=1}^n \rho_{ik} \text{ Need}_i \text{ Need}_k \div \text{Base} \right) \times \text{Risk Dollars}$
 - = A percentage of total risk dollars
- **Now We Have to Calculate the "Need" of Each WBS Element**

Triangular Cost Distribution



- **Probability Density Function**
- **Three Parameters L , M , H Completely Specify Distribution**
- **Mean, Median, Mode, Sigma, All Percentiles can be Expressed in Terms of L , M , and H**

Expected Value and Percentiles of Triangular Distributions

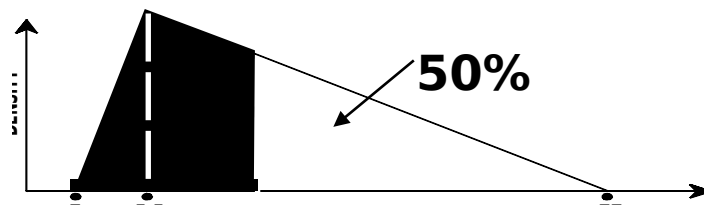
- **Expected Value** $\bar{e} = \frac{L + M + H}{3}$
- **p^{th} Percentile = T_p**
= Dollar Value at which $\left. \begin{matrix} p \\ \text{Cost} \leq T_p \end{matrix} \right\} = p$

$$= L + \sqrt{p(M - L)(H - L)} \quad \text{if } p \leq \frac{M - L}{H - L}$$

$$= H - \sqrt{(1 - p)(H - L)(H - M)} \quad \text{if } p \geq \frac{M - L}{H - L}$$

Example: 50th and 65th Percentiles of Usual Triangular Distributions

- WBS Elements that May Very Well “Need” Risk Dollars at Both the 50th and 65th Percentile Levels Have Triangular Distributions Shaped Like This:



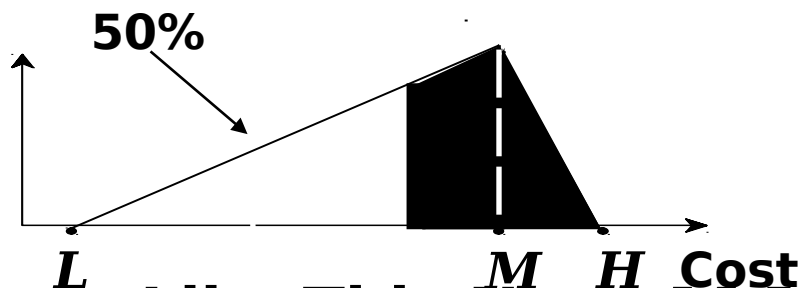
- Distributions Like This Have $M-L < (.50)(H-L)$ and, of course, $M-L < (.65)(H-L)$. Therefore ...
- 50th Percentile = $T_{.50} =$
- 65th Percentile = $T_{.65} = H - \sqrt{(.50)(H-L)(H-M)}$
 $H - \sqrt{(.35)(H-L)(H-M)}$

WBS-Element "Need" Calculated for the Case $M-L < (.50)(H-$

- **At the 50th-Percentile Level ...**
 - "Need" > 0 Always if the Most Likely Cost is Taken as the Point Estimate
 - "Need" = 50th- Percentile Element Cost, Minus Most Likely Element Cost
 - "Need" = $H - \sqrt{(.50)(H - L)(H - M)} - M$
- **At the 65th-Percentile Level ...**
 - Again, "Need" > 0 Always in this Situation
 - "Need" = 65th- Percentile Element Cost, Minus Most Likely Element Cost
 - "Need" = $H - \sqrt{(.26)(H - L)(H - M)} - M$

Example: 50th and 65th Percentiles of Other Triangular Distributions

- WBS Elements that Probably Don't "Need" Risk Dollars at the 50th, but Might or Might Not "Need" Them at the 65th Percentile Level, Have Triangular Distributions Shaped Like This:



- If Distributions Like This Have $M-L > (.65)(H-L)$, then the 65th Percentile $L + \sqrt{(.65)(5)(M-L)(H-L)}$
- If Distributions Like This Have $M-L < (.65)(H-L)$, then the 65th Percentile $H - \sqrt{(.35)(5)(H-L)(H-M)}$

WBS-Element "Need" Calculated for the Case $M-L > (.65)(H-L)$

• At the 65th-Percentile Level

- Need = 65th- Percentile Element Cost, Minus Most Likely Element Cost
- "Need" $= L + \sqrt{(.65)(M-L)(H-L)} - M$
- If "Need" < 0 , We Set Need = 0
- If "Need" > 0 , We Use that Positive Number as the Need of the Element

• At the 50th-Percentile Level ...

- It Must Be True that $M-L > (.50)(H-L)$ also, so We Apply the Following Formula ...
- "Need" $= L + \sqrt{(.50)(M-L)(H-L)} - M$
- This Number is Always Negative in the Situation Described, so We Set Need = 0



"Need" Calculated In Cases when $M-L > (.50)(H-L)$, but $M-L <$

• At the 50th-Percentile Level ...

- "Need" = 50th- Percentile Element Cost, Minus Most Likely Element Cost
- "Need" = $L + \sqrt{(.50)(M-L)(H-L)} - M$
- This Number is Always Negative in the Situation Described Because $(.50)(M-L)(H-L) > (.50)^2(H-L)^2$
Means that $L + (.50)(H-L) - M = (.50)(H-L) - (M-L) < 0$
- Therefore We Set Need = 0

• At the 65th-Percentile Level

- "Need" = 65th- Percentile Element Cost, Minus Most Likely Element Cost
- "Need" = $H - \sqrt{(.35)(H-L)(H-M)} - M$
- If "Need" < 0, We Set Need = 0
- If "Need" > 0, We Use that Value as the Need of the Element



Total Amount of Risk Dollars Needed for 50% Confidence (says Crystal Ball®)

- **50thRisk\$ = 50th-Percentile Total Cost, Minus “Roll-Up” Point Estimate**
- **50thRisk\$ = 1,739.86-1,250.00 = 489.86**

- **Rec of Soft**

WBS Element	M (Mode)
#1	380.000
#2	192.000
#3	76.000
#4	18.000
#5	154.000
#6	58.000
#7	22.000
#8	120.000
#9	230.000
Sum =	1250.000

Percentile (Confidence Level)	Cost Value
5%	1,320.16
10%	1,399.64
15%	1,460.09
20%	1,508.89
25%	1,554.15
30%	1,592.97
35%	1,629.75
40%	1,662.51
45%	1,700.33
50%	1,739.86
55%	1,777.18
60%	1,818.64
65%	1,857.85
70%	1,898.94
75%	1,947.32

l®

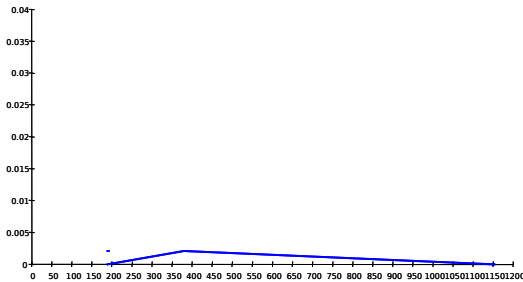


Allocation of System X Risk Dollars to WBS Elements at 50th Percentile

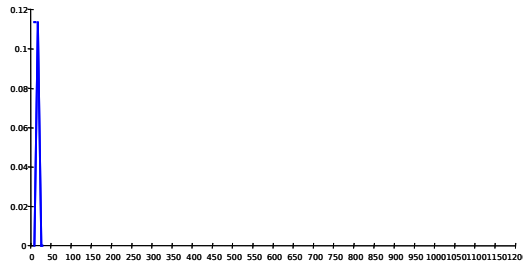
WBS Element	Point Estimate	Allocation as % of Base	Risk-Dollar Allocation	50th Percentile Estimate
#1	380.000	40.264%	197.237	577.237
#2	192.000	17.784%	87.116	279.116
#3	76.000	1.177%	5.765	81.765
#4	18.000	0.000%	0.000	18.000
#5	154.000	13.780%	67.502	221.502
#6	58.000	0.000%	0.000	58.000
#7	22.000	1.503%	7.361	29.361
#8	120.000	0.000%	0.000	120.000
#9	230.000	25.492%	124.877	354.877
Sums =	1,250.000	100.000%	489.857	1,739.857



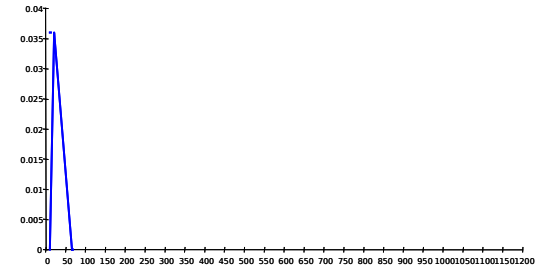
Recall: System X WBS- Element Triangular Cost Distributions



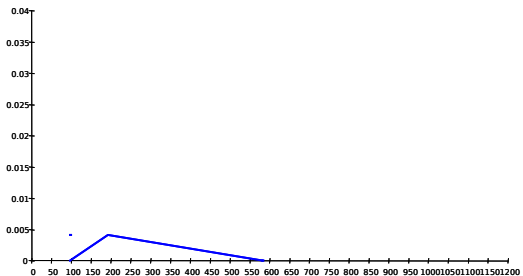
1. Antenna



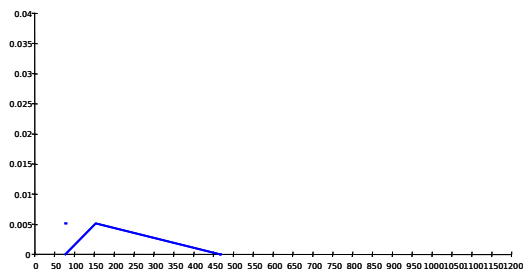
4. Facilities*



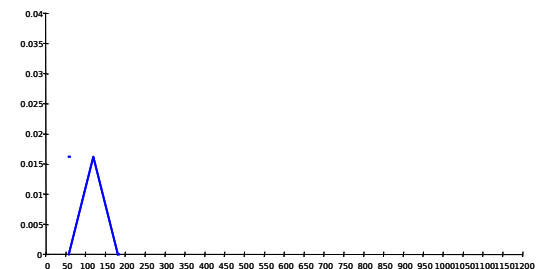
7. Environmental Control



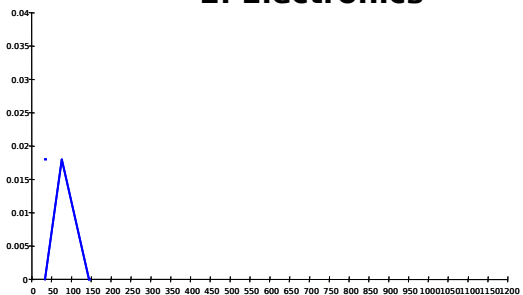
2. Electronics



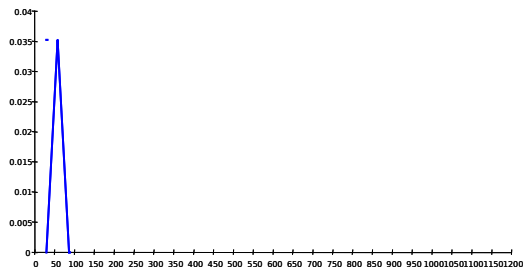
5. Power Distribution



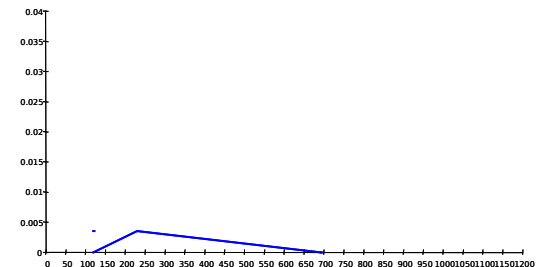
8. Communications



3. Platform



6. Computers



9. Software

* Vertical scale (probability density) different in this graph only.



Total Amount of Risk Dollars Needed for 65% Confidence (says Crystal Ball®)

- **65thRisk\$ = 65th-Percentile Total Cost, Minus “Roll-Up” Point Estimate**
- **65thRisk\$ = 1,857.85-1,250.00 = 607.85)**
- **Recall Output of Crystal Ball® Software:**

WBS Element	M (Mode)
#1	380.000
#2	192.000
#3	76.000
#4	18.000
#5	154.000
#6	58.000
#7	22.000
#8	120.000
#9	230.000
Sum =	1250.000

Percentile (Confidence Level)	Cost Value
5%	1,320.16
10%	1,399.64
15%	1,460.09
20%	1,508.89
25%	1,554.15
30%	1,592.97
35%	1,629.75
40%	1,662.51
45%	1,700.33
50%	1,739.86
55%	1,777.18
60%	1,818.64
65%	1,857.85
70%	1,898.94
75%	1,947.32



Allocation of System X Risk Dollars to WBS Elements at 65th Percentile

WBS Element	Point Estimate	Allocation as % of Base	Risk-Dollar Allocation	65th Percentile Estimate
#1	380.000	39.125%	237.819	617.819
#2	192.000	17.269%	104.967	296.967
#3	76.000	1.853%	11.262	87.262
#4	18.000	0.165%	1.000	19.000
#5	154.000	13.423%	81.593	235.593
#6	58.000	0.556%	3.381	61.381
#7	22.000	1.478%	8.987	30.987
#8	120.000	1.346%	8.184	128.184
#9	230.000	24.785%	150.653	380.653
Sums =	1,250.000	100.000%	607.845	1,857.845



Not So Fast! There's More to Discuss

- **Statistical Fact: Actual WBS-Element 50th Percentiles Do Not Sum to the 50th Percentile of Total Cost**
- **But Our (so-called) “50th Percentile Estimates” Really Do Sum to the 50th Percentile of Total Cost**
- **Why?**
 - **Because We Calculated the 50th Percentile of Total Cost First**
 - **Then We Divided the 50th Percentile Total-Cost Among the WBS Elements in Proportion to their Riskiness, with Inter-Element Correlations Taken into Account**
- **Therefore the Numbers You See Summing to the 50th Percentile of Total Cost are NOT the Actual 50th Percentiles of Each of the WBS Elements**
- **Same Assertions Hold True for 65th and All Other Cost Percentiles**

Note: Actual Percentiles Represented by Term "50th Percentile Estimates"

WBS Element	L	M (Point Estimates)	H	"50th Percentile Estimates"	$\frac{M-L}{H-L}$	True Percentiles of So-Called "50th Percentile Estimates"	Actual 50th Percentile Estimates
#1	191.000	380.000	1151.000	577.237	0.197	55.52%	542.658
#2	96.000	192.000	582.000	279.116	0.198	51.60%	274.153
#3	33.000	76.000	143.000	81.765	0.391	49.12%	82.296
#4	9.000	18.000	27.000	18.000	0.500	50.00%	18.000
#5	77.000	154.000	465.000	221.502	0.198	50.86%	219.370
#6	30.000	58.000	86.000	58.000	0.500	50.00%	58.000
#7	11.000	22.000	66.000	29.361	0.200	44.53%	31.215
#8	58.000	120.000	182.000	120.000	0.500	50.00%	120.000
#9	120.000	230.000	691.000	354.877	0.193	57.08%	328.211
Sums =		1250.000		1739.857	p = .50	50.00%	1673.903

*p*th Percentile of a WBS Element

$$= L + \sqrt{p(M-L)(H-L)} \quad \text{if } p \leq \frac{M-L}{H-L}$$

$$= H - \sqrt{(1-p)(H-L)(H-M)} \quad \text{if } p \geq \frac{M-L}{H-L}$$

Actual Percentiles Represented by "65th Percentile Estimates"

WBS Element	L	M (Point Estimates)	H	"65th Percentile Estimates"	$\frac{M-L}{H-L}$	True Percentiles of So-Called "65th Percentile Estimates"	Actual 65th Percentile Estimates
#1	191.000	380.000	1151.000	617.819	0.197	61.59%	642.025
#2	96.000	192.000	582.000	296.967	0.198	57.14%	324.436
#3	33.000	76.000	143.000	87.262	0.391	57.85%	92.211
#4	9.000	18.000	27.000	19.000	0.500	60.49%	19.470
#5	77.000	154.000	465.000	235.593	0.198	56.39%	259.491
#6	30.000	58.000	86.000	61.381	0.500	61.35%	62.574
#7	11.000	22.000	66.000	30.987	0.200	49.34%	36.897
#8	58.000	120.000	182.000	128.184	0.500	62.33%	130.127
#9	120.000	230.000	691.000	380.653	0.193	63.41%	387.469
Sums =		1250.000		1857.845	p = .65	65.00%	1954.700

Recall List of Total-Cost Percentiles

- **Output of Crystal Ball® Software:**

Percentile (Confidence Level)	Cost Value
5%	1,320.16
10%	1,399.64
15%	1,460.09
20%	1,508.89
25%	1,554.15
30%	1,592.97
35%	1,629.75
40%	1,662.51
45%	1,700.33
50%	1,739.86
55%	1,777.18
60%	1,818.64
65%	1,857.85
70%	1,898.94
75%	1,947.32
80%	1,999.56

1673.90
(42nd Percentile)

1954.70
(76th Percentile)

Observations on the Risk-Dollar Allocation Process

- **Under our Definition of “Need”, Percentage of Risk Dollars Allocated to Each WBS Element Depends on our Choice of “Point” Estimate of Total Cost**
 - **“Point” Estimate Can be Defined as**
 - “Roll-Up of Element Most Likely Costs”
 - “Expected Cost”
 - “50th-Percentile Cost”
 - “Most Likely Total Cost”
 - ... or Whatever
 - **An Element’s “Need” is Based on**
 - Its Point Estimate (however that is defined)
 - Its Risk Characteristics (i.e, skewness of its cost distribution)
 - Correlation of its Risks with other WBS-Elements’ Risks
- **Risk-Dollar Percentage Allocated to Each Project Element Also Depends on Level of Confidence (50th, 65th, etc.) Considered Appropriate for Management Reserve**

Summary

- **Allocation of Risk Dollars to WBS Elements Offers Supporting Justification when Requesting Management Reserve**
- **Before Deciding How to Allocate Dollars, Analysts Must ...**
 - ... Assign WBS-Element Cost Probability Distributions
 - ... Calculate Total-Cost Probability Distribution
 - ... Agree upon Meaning of "Point Estimate" of Total Cost
 - ... Agree upon Confidence Level Required for Risk Coverage
 - ... Agree upon Specifications for Allocation Decision
- **How This Risk-Dollar Allocation Procedure Works**
 - Define Dollar-Valued "Need" of Each WBS Element
 - Calculate Dollar-Valued "Need" of Each WBS Element, Taking into Account Inter-Element Correlations
 - Sum All "Needs" to Obtain Total "Need Base"
 - Allocate Risk Dollars to WBS Elements in Proportion to their Fractions of the Need Base
 - You Don't Have to Worry About Someone in the Audience Noticing that Your 65th Percentile Estimates "Don't Add Up"



Acronyms

<i>H</i>	High (Most Pessimistic) Cost
<i>L</i>	Low (Most Optimistic) Cost
<i>M</i>	Most Likely Cost
PE	Point Estimate
WBS	Work- Breakdown Structure